or

$$\frac{\partial}{\partial t}(\gamma u) = jE + \left\{\frac{\partial \varepsilon}{\partial t}\frac{E^2}{8\pi} - \frac{\partial}{\partial t}\left[T\left(\frac{\partial \varepsilon}{\partial T}\right)_\gamma \frac{E^2}{8\pi}\right]\right\}, \tag{28}$$

which reduces to (2) with ε = const. The equation of continuity of the total current [1] is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(j + \frac{1}{4\pi}\frac{\partial D}{\partial t}\right)\right] = 0, \tag{29}$$

whose first integral is

$$2\pi r^2\left(j + \frac{1}{4\pi}\frac{\partial D}{\partial t}\right) = I(t). \tag{30}$$

The system (28) and (30) is closed by Ohm's law (1) and the condition of potentiality $E = -\partial\varphi/\partial r$. Integration of this system is complicated by the fact that ε and σ depend on T (for a solid it may be assumed that γ = const). Equation (27) differs from the analogous relation in [1] by a redefinition of the internal energy and Poynting's vector.

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MEASUREMENT OF THE TOTAL SCATTERING CROSS SECTIONS OF INERT GASES IN THE RELATIVE ENERGY RANGE 7-17 eV

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The description of collisional processes in rarefied gases requires information about the interaction potentials. Theoretical and experimental data have now been accumulated on the short-range intermolecular forces. The status of this field is reviewed in [1-4]. The chief results were obtained primarily from experiments on scattering of high-energy beams (E ~ 1 keV) by small angles ($\theta \sim 10^{-2}$ rad) or from measurements of the attenuation of beams passing through a layer of scattering gas (gas target). Empirical values of the parameters in power-law and exponential potentials for different atomic and molecular gases are collected together in [1].

The region of moderate interaction energies (~10 eV) has been less studied, since the question of producing monoenergetic neutral particle beams in the region 1-10 eV has yet to be resolved. In the last few years, problems for which reliable information about the interaction potentials in this energy range is required in order to obtain qualitative and quantitative results have appeared. They include the questions of the formation of the characteristic external atmosphere (CEA) around aircraft at high altitudes. One of the chief mechanisms for transport of pollutants to the sensitive elements of the environment in the formation of CEA are return flows, determined by collisions between the pollutant particles and the particles of the incident flow.

In this case the problem reduces to summing the particle fluxes on the corresponding surface element dS of the body in the flow [5] $dN = n_1 n_2 d\sigma g_{21} d\tau dS$, etc., where n_1 and n_2 are the particle densities in the incident flow and the products of mass release from the structural surfaces (as a result of desorption, degassing, sublimation, evaporation, etc.) in the volume element of the physical space $d\tau$, $g_{21} = |\mathbf{v}_2 - \mathbf{v}_1|$ is the relative velocity of the colliding particles (the index 1 refers to the particles in the incident flow and the index 2 refers to the mass-release particles); $d\sigma$ is the differential cross section for scattering into the solid angle $d\omega$, at which the element dS can be seen from the center of the volume $d\tau$.

In the case of elastic-sphere molecules the differential scattering cross section in the coordinate system fixed to dS can be represented in the form

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$$d\sigma = \frac{\sigma}{4\pi}\left\{\pm 2t_k \cos\nu + \frac{1 + t_k^2 \cos 2\nu}{\sqrt{1 - t_k^2 \sin^2\nu}}\right\} d\omega. \tag{1}$$

Here $t_k = \frac{m_k(m_1+m_2)}{m_1 m_2}\frac{|g_c|}{|g_{21}|}$; σ is the total scattering cross section for spherical molecules; ν is the angle between the velocity vector of the particle moving in the direction dS after the collision and the velocity vector of the center of mass $g_c = (v_2 + \mu v_1)/(1+\mu)$; $\mu = m_1/m_2$ is the mass ratio of particles of different classes; k = 1, 2.

Thus the return flows to the surfaces monitored depend directly on the total scattering cross section σ.

The kinetics of single-collision rarefied-gas flows is also studied in the problem of shielding optical systems of infrared space telescopes from contamination by condensing gas in the upper atmosphere, primarily, atomic oxygen [6]. The flow of oxygen atoms can be significantly attenuated by blowing noncondensing gases (Ne, He) over the inner surfaces of the telescopes. The effect of the differential transverse scattering cross sections of the injected and condensing gases on the process of formation of contaminants is analyzed in [6].

The question of the choice of model for the interaction of molecules is not simple, and often requires special intuition in order to obtain results acceptable for practical applications. The use of any model is justified, if in the absence of information about the collision cross sections of real molecules it gives plausible results and is not too cumbersome to use. In computational practice the hard-sphere model is simplest and most convenient. In this model the diameters of the molecule-spheres are determined, as a rule, from information about the temperature dependence of the coefficient of viscosity. One way to model the required temperature dependence of the coefficient of viscosity is to introduce the "pseudogas" model [7], in which it is assumed that the collision cross section is a function of the relative velocity g_{21}, while the collision itself occurs according to the law of hard spheres.

In this paper we present the results of an experimental determination of the total cross section for scattering of a molecular beam with energy ~10 eV by a gas target. The beam is produced by charge exchange of ions in the jet of an induction plasma accelerator with a magnetic nozzle, employed previously for aerodynamic measurements [8]. Measurements performed by the time-of-flight (phase) method showed that the accelerator gives a neutral flow of Ar, Kr, and Xe with energy E = 7-17 eV and intensity not less than $N = 10^{18}$ $m^{-2}\cdot sec^{-1}$. This energy range for Ar atoms corresponds to a flow velocity v = 6-9 km/sec.

The traditional method for determining total scattering cross sections experimentally is based on the measurement of the relative intensity of the beam passing through the target [1]. The experimental arrangement consists of the following. Let a monovelocity beam of particles pass through a gas target with thickness dx. If it is assumed that the flux density of the particles in the beam equals N, then after passing through the layer dx its intensity will be N − dN. The relative change in the flux density is dN/N = σndx (n is the number density of target particles per unit volume).

Integrating this equation we obtain

$$N = N_0 \exp(-n\sigma l), \tag{2}$$

where N_0 is the flux density of particles in the beam in the section x = 0, and N is the flux density in the section x = ℓ. The formula (2) shows that the total scattering cross section σ can be measured experimentally, if N and N_0 have been measured, i.e.,

$$\sigma = (1/nl)\ln(N_0/N). \tag{3}$$

The experiment can be altered somewhat by measuring N_0 and N not in different sections, but rather in one and the same section of the jet ℓ but with different pressures in the working chamber. Then N_0 is the flux density of particles in the section ℓ with a low pressure in the working chamber, when the average mean-free path length of the particles in the beam amongst the target particles $\lambda_{21} \gg \ell$, while N is the flux density in the section ℓ with a pressure such that $\lambda_{12} \geqslant \ell$. However, there are certain difficulties in making a reliable measurement of the intensity of the molecular beam with high particle concentrations in the working chamber.

In this work we measured the total scattering cross sections of inert gas atoms not based on the loss of intensity, but rather based on the loss of momentum flux F_0/F of the molecular beam traversing a distance ℓ. In this case the formula (3) has the form

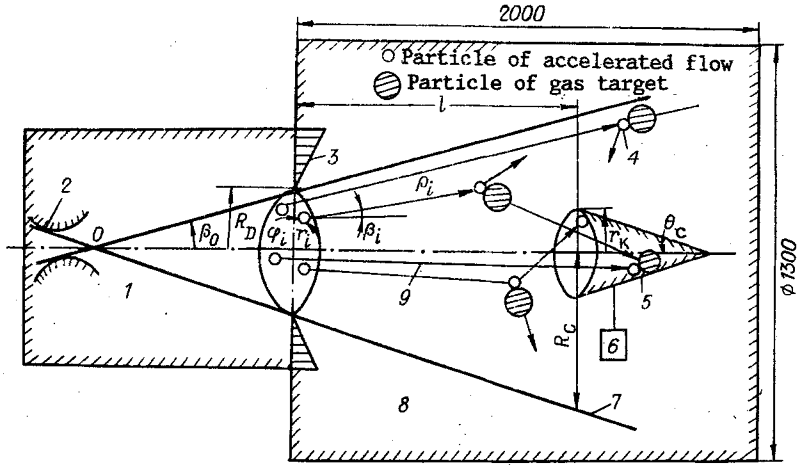

Fig. 1

$$\sigma = (1/nl)\ln[(F_0/F)f(\theta)] \tag{4}$$

[$f(\theta) = K_1K_2$ is an instrumental function characterizing the efficiency with which the particles scattered by an angle θ are recorded]. The coefficient K_1 takes into account the divergence of the flux, while K_2 takes into account the finite dimensions of the detector, the scattering target, and the beam cross section.

This approach has appreciable advantages over the above-described traditional method, since based on the total effect of the momentum loss the gas modeled by hard spheres is equivalent to a real gas [9]. This last remark enables using the results obtained for calculations. The total cross section σ, appearing in (1), for the system of interacting gases under study should correspond to fixed interaction energies and can be taken from experiment.

The experimental arrangement is shown in Fig. 1, where 1 is the chamber of the source, 2 is the magnetic nozzle, 4 is the particle collision, 7 is the boundary of the jet, and 9 is collisionless motion. Torsion balances 6 with a total-pressure attachment 5 installed on them, consisting of a hollow cone with a half-angle $\theta_C = 12°$ and a base radius $r_C = 25$ mm, were placed at a base distance $\ell = 800$ mm from the input diaphragm 3. In this case the attachment played the role of a detector. The radius of the core of the flow at the location of the attachment $R_C \approx 75$ mm. The deflection of the balances φ_0 was measured for a residual gas pressure of $\sim 6\cdot 10^{-4}$ Pa in the working chamber 8. Then the residual-gas pressure was increased by additional injection of gas, employed as the target, up to values of $3\cdot 10^{-3}$ Pa in such a way that the mean-free path length of the particles in the beam on particles of the gas target would not be less than the baseline distance

$$\lambda_{12} \geqslant l, \tag{5}$$

and the deflection of the balances φ was measured once again. In the entire series of experiments the value of λ_{12} was measured in the range 3ℓ-5ℓ.

The particle concentration in the working chamber was determined from the pressure, measured with the help of a VIT-2 vacuum meter. The ratio of the quantities measured by the balances is the experimental value of the loss of momentum flux when the accelerated flux is scattered by the gas target

$$F_0/F = \varphi_0/\varphi. \tag{6}$$

The induction accelerator operated on argon, and Ne, Ar, Kr, and Xe were employed as the gas target.

The instrumental function appearing in the expression (4) was found by means of a direct Monte Carlo numerical simulation of the process of scattering of particles in the jet by particles in the target with subsequent use of the iteration method to solve Eq. (4). The problem was solved on the basis of the theory of "first molecular collisions" [9]. Particles of the beam, traversing a distance ℓ without collisions, as well as particles of both classes singly scattered over this distance were separated amongst the particles recorded by the attachment. The characteristic Knudsen number $Kn_{ij} = n_2\sigma_{12}r_C/2$ for the particles — taking into account (5) — fell into the range 50-80. In this case subsequent collisions of the scattered particles with particles of the background gas can be neglected [9].

TABLE 1. Interaction of a System of Particles

Interacting system of particles	$\sigma \cdot 10^{20}$, m^2	$\sigma_0 \cdot 10^{20}$, m^2
Ar—Ne	17	31
Ar—Ar	25	42
Ar—Kr	30	50
Ar—Xe	32	58

The numerical modeling algorithm for determining the coefficients K_1 and K_2 consists of the following. The particles start in the plane of the output section of the diaphragm with radius R_D, separating the working chamber from the source chamber (see Fig. 1). It is presumed that the flux corresponds to flow from a spherical source with a pole at the point O.

The coordinates of the starting point r_i, φ_i evolve according to the formulas $r_i = R_D \tan\beta_i/\tan\beta_0$, $\varphi_i = 2\pi\xi_{i+1}$, where $\cos\beta_i = \xi_i(1+\cos\beta_0)+\cos\beta_0$; ξ_i, ξ_{i+1} are random numbers distributed uniformly over the interval [0, 1].

We denote by the index 1 the particles of the beam and by the index 2 the particles of the background gas. Then the probability density for the mean-free path ρ of a beam particle amongst particles in the gaseous target in the working chamber is given by the relation

$$P(\rho) = (1/\lambda_{21})\exp(-\rho/\lambda_{12}). \tag{7}$$

Here $\lambda_{12} = 1/n_2\sigma_{12}$ is the mean-free path length of beam particles amongst particles in the gaseous target; σ_{12} is the scattering cross section sought for particles of different classes. For the zeroth-order approximation we take the scattering cross section for the gas at rest $\sigma_{12}^0 = \pi[(D_1 + D_2)/2]^2$ (D_1 and D_2 are the nominal diameters of the spherical molecules). The coordinate of the point of collision along the corresponding stream line is determined according to the probability density (7) from the formula $\rho_i = -\lambda_{12}\ln\xi_{i+2}$. If the collision occurred at a distance less than ℓ, then the velocities of the collision partners are determined at the point determined by the radius vector ρ_i (it is assumed that the particles of the background gas are at rest, i.e., $\mathbf{v}_2 = 0$): $\mathbf{v}_1' = \mathbf{v}_1 - 2M_2(\mathbf{v}_1\mathbf{K}_{12})\mathbf{K}_{12}$, $\mathbf{v}_2' = -2M_1(\mathbf{v}_1\mathbf{K}_{12})\mathbf{K}_{12}$, where $M_1 = m_1/(m_1 + m_2)$ and $M_2 = m_2/(m_1 + m_2)$ are the reduced masses of the colliding particles; $\mathbf{K}_{12}$ is the unit vector of the line of centers of the colliding particles, and its direction is given by the angles ν and μ and is determined from the formulas $\sin\nu_i = \sqrt{\xi_{i+3}}$, $\mu_i = 2\pi\xi_{i+4}$.

In all cases of particles striking the surface of the attachment the molecular properties carried by them were summed. After a series of tests $i = 1,\ldots,L$ was performed, the coefficients K_1 and K_2 for one iteration cycle were determined from the relations

$$K_1 = \frac{\sum_{i=1}^{L_0} v_{1xi} \sum_{j=1}^{L_1} v_{1xj}^2}{\sum_{j=1}^{L_1} v_{1xj} \sum_{i=1}^{L_0} v_{1xj}^2}; \tag{8}$$

$$K_2 = (P_1 + P_2 + P_3)/P_0. \tag{9}$$

Here v_{1x} is the axial component of the velocity of the molecular beam; P_0 and P_1 are the momenta transferred by the beam particles, traversing a distance ℓ without collision in the free molecular regime and in the presence of the background gas; P_2 and P_3 are the momenta transferred by the collision partners:

$$P_0 = \sum_{i=1}^{L_0} m_1(v_{1x} + v_{1xw})_i, \quad P_1 = \sum_{i=1}^{L_1} m_1(v_{1x} + v_{1xw})_i,$$
$$P_2 = \sum_{i=1}^{L_2} m_1(v_{1x}' + v_{1xw}')_i, \quad P_3 = \sum_{i=1}^{L_3} m_2(v_{2x}' + v_{2xw}')_i. \tag{10}$$

In the expressions (8) and (10) L_0 and L_1 are the number of particles traversing a distance ℓ without collision; L_2 and L_3 are the number of particles of both classes reaching the attachment after collisions. In the summation of the momentum fluxes in (10) it was assumed that the particles reaching the inner surface of the attachment are reflected diffusely with a Maxwellian velocity distribution v_w with a temperature T_w = 300 K. The statistical error in the calculations did not exceed 2% for a total number of tests $L = 10^5$.

After expression (6) is substituted into (4), taking into account (8) and (9), the value of the total scattering cross section at the first section at the first iteration was determined and the process of collision of the beam particles with the particles of the gas target was played out again. As the calculation showed, 3-4 iterations were sufficient for the process to converge.

The values of the total cross sections σ for scattering by different gases, found in the experiment, are given in Table 1; they are approximately 1.5-2 times smaller than the corresponding values σ_0 for the gas at rest, which corresponds to the essence of the phenomenon. The nominal molecular diameters for different gases for calculating σ_0 were taken from [10].

As an example we shall compare the results obtained with the results of calculations based on Sutherland's formula. In our case the energy of the molecules in the beam $E \sim 10$ eV, which corresponds to a gas temperature $T \sim 10^5$ K. For these temperatures Sutherland's formula has the form [11] $\sigma = \sigma_0 273/(273 + C)$. For argon $C = 142$ and $\sigma_0 = 1.5\sigma$. Thus the data in Table 1 agree well with the values found from Sutherland's formula in the limit $T \to \infty$.

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